ALGORITHM FOR ONLINE ESTIMATION OF POWER SYSTEM PARAMETERS

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR

THE DEGREE OF

MASTER OF TECHNOLOGY

IN

ELECTRICAL ENGINEERING

By

ROSY PRADHAN

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NATIONAL INSTITUTE OF TECHNOLOGY, ROURKELA

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UNDER THE GUIDANCE OF

PROF. P.K. RAY

DEPARTMENT OF ELECTRICAL ENGINEERING

NATIONAL INSTITUTE OF TECHNOLOGY, ROURKELA

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ROURKELA

CERTIFICATE

*This is to certify that the thesis entitled, "**ALGORITHM FOR ONLINE ESTIMATION OF POWER SYSTEM PARAMETER**" submitted by* ***Rosy Pradhan*** *in partial fulfillment of the requirements for the award of Master of Technology Degree in Electrical Engineering with specialization in "**Control & Automation"** at National Institute of Technology, Rourkela is an authentic work carried out by her under my supervision and guidance. To the best of my knowledge, the matter embodied in this Project review report has not been submitted to any other university/ institute for award of any Degree or Diploma.*

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ACKNOWLEDGEMENT

I express my deepest gratitude to my project guide **Prof. P.K. Ray** whose encouragement, guidance and support from the initial to the final level enabled me to develop an understanding of the subject.

Besides, we would like to thank to **Prof. A.K. Panda**, Head of the Electrical Engineering Department, National Institutes of Technology, Rourkela for providing their invaluable advice and for providing me with an environment to complete our project successfully.

I am deeply indebted to all faculty members of Electrical Engineering Department, National Institutes of Technology, Rourkela, for their help in making the project a successful one.

Finally, I take this opportunity to extend my deep appreciation to my **family** and **friends**, for all that they meant to me during the crucial times of the completion of my project.

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Date: 22.05.2013

Place: Rourkela

CONTENTS

ABSTRACT:

Power system parameters are having great contribution in the operating, monitoring and controlling of electric device. Frequency and harmonics are the two vital parameters which influence different relay functionality of power system. This study was made to estimate the frequency and harmonics of measuring voltage or current signal in presence of random noise and distortion. Here we are using complex least mean square (LMS), nonlinear least square (LS) and recursive least square (RLS) algorithm for measuring the parameter from the distorted voltage signal. The performances of these algorithms are studied through simulation and the experimental setup has been made to evaluate the robustness of the above algorithms.

LISTS OF FIGURES:

ABBREVIATIONS:

DFT: Discrete Fourier Transformation

DSO: Digital Storage Oscilloscope

FFT: Fast Fourier Transformation

LMS: Least Mean Square

LS: Least Square

RLS : Recursive Least Square

RL load: Resistive- Inductive Load

CHAPTER-1

INTRODUCTION

1.1Background:

In electrical power system, frequency and harmonics are the two distinguish operating parameters which are required to remain constant because they reflect the whole situation of the system. Frequency can show the dynamic energy balance between load and generating power where harmonics can constitute the state of the system. Therefore these parameters are regarded as indices for operating power systems in practical. But due to noise, sudden appearance of load –generation mismatches and increasing use of nonlinear load, the frequency of operating system is not constant which is expected in power system environment. As we know depending upon the load condition, the frequency of operation can be taken into consideration over a small allowable range from its standard value. When there is a deviation in system frequency from its nominal value results in change of component reactance which influences different relay functionality of power system. So frequency play a vital role in operating, monitoring and controlling of any power system device. Basically the available frequency estimation techniques are used digitalized samples of voltage or current signal. Frequently, the voltage signal is used for estimation of system parameters because it is less distorted than the line current. Considering the power system voltage signal as purely sinusoidal, the time between two zero crossing is given the system frequency. However in reality, the measured signals are available in distorted form and thus numerous techniques are available for frequency estimation. Zero crossing technique, discrete Fourier transform, least square error, Kalman filtering, orthogonal finite impulse response filtering and iterative approaches are some of the technique in this area. Soft computing technique, neural network and genetic algorithm are also use for power system frequency estimation.

Accurate harmonics estimation is a very important task in power system environment. This estimation is useful for an efficient design of compensatory filter and for characterization of electrical device under non sinusoidal conditions. Therefore it needed continuously monitoring. Most of the cases harmonics comes from the sources which are dynamics in nature and producing time varying amplitudes in generated signal, so it is difficult to estimate the harmonics. Therefore, accurate and fast estimation of amplitude and phase of these frequency components are needed. The most commonly used classical technology for estimation of harmonics is Fast Fourier Transformation (FFT) of the signal and kalman filtering.

This paper represented estimation of frequency and harmonics from a distorted voltage waveform. The distortion of the signal is further enhanced by considering at different situation of power system. The algorithm used here are complex least mean square (LMS) using three phase voltage signal, nonlinear LS estimator and RLS estimator. The first two estimators use batch processing and third one is online processing. An experimental setup was proposed to test the above algorithm and some test results are presented in this study.

1.2Literature review:

1.2.1 Review of power system frequency estimation:

A prony's method with digital algorithm has been proposed by T.Lobos and T.Rezmer et al [1] to estimate the power system frequency in the year of 1997. In this approach at first the distorted voltage signal is filtered using fourier technique algorithm and the coefficient of the filters are calculated by assuming constant frequency. Because of deviation in power system frequency the filter coefficient is not exact. For improving the filter effect here hamming and Blackman window are used where blackman window is worked well in this approach. After that the output signal of the filter is processed using prony's estimation method to calculate the system frequency. The proposed algorithm was tested on computer by assuming the frequency deviation up to 2Hz in presence of higher harmonics.

P.K.Das et al [2] implemented Extended Complex Kalman Filter (ECKF) to calculate frequency from distorted power signal in 1999. In this paper they considered a discrete value of 3-phase voltage signal of power system and converted that discrete signal into complex voltage vector form with the help of well known $\alpha\beta$ transformer. A nonlinear state space is formulated from the complex voltage vector which is further computed to true state of model iteratively by using Extended Complex Kalman Filter (ECKF) with significant noise and harmonics distortion. The speed of convergence of this technique is reduced by 3 cycles and this can be improved significantly if we considered the harmonics in the state space formulation. Here the estimated frequency error is close to .01Hz to .02Hz in presence of noise. This approach is worked well for decay or rise and step change in frequency. From this proposed technique we get the idea of different situation of power system.

A complex form of Least Mean Square (LMS) algorithm is proposed by A.K.Pradhan et al [3] to estimate the power system frequency. A complex form of LMS algorithm is derived by the $\alpha\beta$ transformer. Basically the LMS algorithm is preceded by first computing the error signal which is then used to calculate the updated coefficient. Initially all the coefficient of this algorithm is made to zero. The cycle of updating is continued until we reached a steady state result. In LMS algorithm having poor convergence rate due to fixed step size. To overcome this problem time varying step size is implemented in the following algorithm. The simplicity of the algorithm provided the modest resources for implementation.

A nonlinear Least Square (LS) technique is employed by R. Chudamani, Krishna et al [4] to measuring the frequency of electrical power system. Here estimation of fundamental frequency is carried out over arrange of allowable frequency by performing 1-D search. The measuring voltage signal of this algorithm is modeled by implementing Fourier series. The above technique is very much flexible to estimate the frequency in presence of harmonics either selectively or in total.

1.2.2 Review of power system harmonics estimation:

Maamar Bettayed et al [5] proposed a recursive method to estimate the harmonics of power system. This paper described about the online estimation of harmonics amplitude and phase using several variant of Recursive Least Square (RLS) algorithm. As we know recursive method has simple computation and good convergence properties. Therefore it is easy to implement for online estimation of harmonics in noisy environment. Here all the estimation error is within 3-4% and maximum deviation is within 9-10%. This algorithm is performed well in single frequency estimation as compared to multiple frequency estimation. Because the noise power signal of multiple frequency estimation is larger than the single frequency estimation. Moreover the recursive algorithm produced better results for single frequency signal

Beides and Heydt [6] estimated bus voltage magnitudes and phase angles of the fundamental and higher harmonics from noisy measurement using Kalman Filter method .This algorithm is also tested with IEEE 14 bus system. These results are helpful in studying the total harmonic distortion over a full cycle. It is also helpful in designing power filter to minimize harmonics.

The combination of both Fourier Linear Combiner and Extended Complex Kalman Filter (ECKF) has been proposed in [7] for power system harmonic estimation. Kalman Filter estimates amplitude and phase when frequency is fixed. However, when frequencies vary, it is unable to retune itself to the frequency changes. Similarly, Fourier Linear Combiner, using single layer neural network able to estimate harmonics at static frequency but during frequency change tracking time becomes much larger and there is more error in estimation.

1.3. Motivation of Project work:

As discussed before, electrical power system environment is polluted by random noise, harmonics and reactive power disturbance due to sudden mismatch of generation-load and frequent use of nonlinear load in electrical device. It results deviation of fundamental frequency from its standard value and elevates harmonics level in the power system network which is undesirable. It is a difficult task to estimate the exact frequency and harmonics amplitude and voltage in presence of random noise. Although complex LMS algorithm is used for calculation of power system frequency but attention has been not paid to estimation of frequency in different power system condition, which motivated to estimate to estimate frequency in different situation of power system. As discussed above, nonlinear LS and RLS has been applied to estimate the frequency and harmonics of power system however hardware implementation of these algorithms has not been done in this field. So it was motivated to carry out an experimental set up for estimation of power system frequency and harmonics.

1.4 Objective of the thesis:

The objectives of the thesis are as follows:

- To estimate the frequency using complex LMS algorithm and analyze it for different power system situation, although LMS algorithm is simple to formulate but having the drawback of poor convergence rate. So we considered complex LMS algorithm with time varying step size which overcome the above drawback.

- To analyze the nonlinear LS algorithm for calculation of frequency in a range of frequency.
- To analyze the nonlinear RLS for calculation of frequency and harmonics amplitude and phase.
- To proposed a hardware set up to check the robustness of above algorithm.

1.5 Thesis organization:

Chapter-1 consists of an introduction of power system parameter. It also includes a brief literature review on estimation of power system frequency and harmonics and it focus on the motivation and objective of the project.

Chapter-2 deals with the mathematical analysis of complex LMS, nonlinear LS for calculating power system frequency whereas RLS method is used for estimation of both frequency and harmonics. This chapter also consists of simulation results of each algorithm.

Chapter-3 contains the hardware set up information, which is proposed to evaluate the above algorithm flexibility and robustness. And it also contained the experimental results obtained by the data and their summary.

Chapter-4 deals with the conclusion and suggestion of future work.

Chapter-5 contains the references.

CHAPTER-2

MATHEMATICAL ANALYSIS

2.1 Power system frequency estimation:

2.1.1 Introduction:

A system with no loss performance is taken into consideration in power system environment. For getting better power quality the measured voltage or current signal must be purely sinusoidal in shape. But in practical, it degenerate due to type of source, under voltage, over voltage, variation in frequency and harmonics, nonlinear load and generated load mismatches. Hence there is a need of fast and accurate estimation of supply frequency and voltage for improving the power quality in presence of noise and higher harmonics. Most of the technique foe estimation of power system parameter is used digitized samples of supply voltage. Basically frequency of a system indicates the time between two zero crossing of voltage signal where the voltage signal is purely sinusoidal. However in reality the measured signals are available in distorted form. Hence a numerous method is proposed to calculate the frequency. Discrete Fourier transformation (DFT), least square error, Kalman filtering and iterative approaches (2,6-11) are some of the popular technique in this area. In this chapter complex LMS, nonlinear LS and RLS has been employed to obtain the power system frequency.

2.1.2 Frequency estimation using complex LMS:

The LMS algorithm is initialized by setting all coefficients to zero. Then it proceeds by first computing the error signal which is then used to compute the updated coefficients. The LMS approach of frequency estimation is shown in fig 1 where $\mathbf{U_K}=[\mathbf{U_{0K}} \quad \mathbf{U_{1K}} \quad \cdots \quad \mathbf{U_{(N-1)k}}]^{\mathbf{T}}$ is the input data vector at kth instant, $\mathbf{Y_K}$ is the desired signal and $\widehat{\boldsymbol{Y_K}}$ is the estimated signal. The signal can be estimated correctly with a suitable coefficient $\mathbf{W_K}$, which is obtained by the minimization of the error signal $\mathbf{e_K}$ **.**

At each sample, the weight vector is calculated as

$$\mathbf{W_{K+1}}=\mathbf{W_K}+\boldsymbol{\mu}(-\boldsymbol{\nabla}_{\mathbf{K}}) \qquad \textbf{(2.1)}$$

Where $\boldsymbol{\mu}$ is the adaptation parameter and $\boldsymbol{\nabla_K}$ is the gradient of error performance surface with respect to filter coefficient.

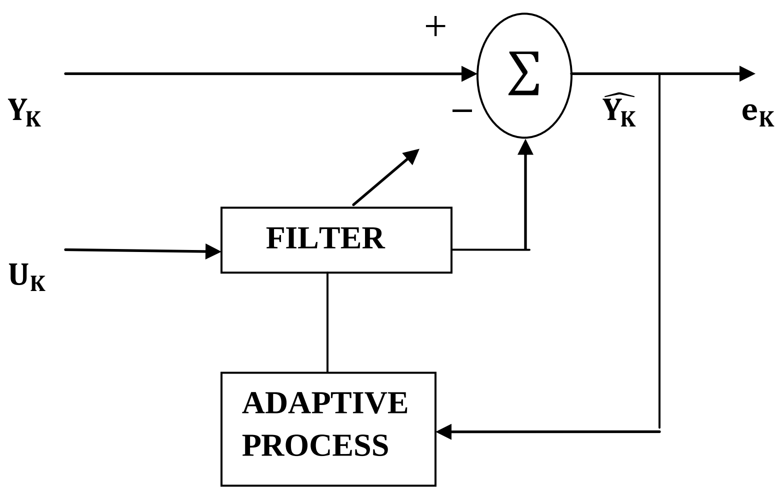


Fig2.1. LMS filter structure

In LMS based algorithm, a three voltage signal is consider as input signal which can be represented in discrete form as follows

$$\mathbf{Y_{ak} = Y_m cos(\omega k\Delta t + \emptyset) + \varepsilon_{ak}}$$

$$\mathbf{Y_{bk} = Y_m cos(\omega k\Delta t + \emptyset - \frac{2\pi}{3}) + \varepsilon_{bk}}$$

$$\mathbf{Y_{ck} = Y_m cos(\omega k\Delta t + \emptyset + \frac{2\pi}{3}) + \varepsilon_{ck}} \quad \textbf{(2.2)}$$

Where $\boldsymbol{\varepsilon_k}$ are the noise terms, $\mathbf{Y_m}$ is the peak amplitude of the fundamental component, $\boldsymbol{\Delta t}$ is the sampling time period, $\mathbf{k}$ is the sampling instant, $\emptyset$ is the phase of the fundamental component and $\boldsymbol{\omega}$ is the angular frequency of the voltage signal. By $\boldsymbol{\alpha\beta}$ transform, the three phase voltage signal is converted to a complex signal form as follows.

$$\begin{bmatrix} \mathbf{Y_{\alpha k}} \\ \mathbf{Y_{\beta k}} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \mathbf{1} & -\frac{1}{2} & -\frac{1}{2} \\ \mathbf{0} & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} [\mathbf{Y_{ak} \quad Y_{bk} \quad Y_{ck}}]^{\mathbf{T}} \quad \textbf{(2.3)}$$

A complex voltage $\mathbf{Y_k}$ is obtained from (3) as

$$\mathbf{Y_k = Y_{\alpha k} + jY_{\beta k}}$$

$$= \mathbf{Y e^{j(\omega k \Delta t + \varphi)} + \eta_k}$$

$$= \mathbf{\widehat{Y_k} + \eta_k} \tag{2.4}$$

Where $\mathbf{Y}$ is the amplitude of the complex signal $\mathbf{Y_k}$ and $\mathbf{\eta_k}$ is the noise component**.**

The error signal $\boldsymbol{\epsilon}_\mathbf{k}$ in this case is

$$\boldsymbol{\epsilon}_\mathbf{k} = \mathbf{Y_k - \widehat{Y_k}} \tag{2.5}$$

Where $\mathbf{\widehat{Y_k}}$ is the estimated voltage value at kth instant which is further modeled as

$$\mathbf{\widehat{Y_k} = W_{k-1}\widehat{Y_{k-1}}} \tag{2.6}$$

Where$\mathbf{W_k} = \boldsymbol{\epsilon}^{\mathbf{j(\widehat{\omega_{k-1}\Delta t})}}$, and $\boldsymbol{\widehat{\omega}}$ is the estimated angular frequency.

The algorithm minimizes the complex weight vector $\mathbf{W_k}$ at each instant as

$$\mathbf{W_k = W_{k-1} + \mu_k e_k \widehat{Y_k^*}} \tag{2.7}$$

Where $*$ represents the complex conjugate of the value and $\boldsymbol{\mu}$ is the convergence factor controlling the stability which is update as

$$\mathbf{\mu_{k+1} = \lambda\mu_k + \gamma p_k p_k^*} \tag{2.8}$$

Where $\mathbf{p_k}$ is the cross correlation of $\mathbf{e_k}$ and $\mathbf{e_{k-1}}$ and is computed as

$$\mathbf{p_k = r p_{k-1} + (1 - r) e_k e_{k-1}} \tag{2.9}$$

Where $\mathbf{r}$ is the exponential weighting parameter and$(\mathbf{0} < r < \mathbf{1})$,$\boldsymbol{\lambda}(\mathbf{0} < \lambda < 1)$ and $\boldsymbol{\gamma} > \mathbf{0}$ control the convergence time.$\mathbf{\mu_{k+1}}$ is set to $\mathbf{\mu_{max}}$ or $\mathbf{\mu_{min}}$ when it falls below or above the lower and upper boundaries respectively.

Then at each sampling interval, the frequency is calculated as

$$\mathbf{\widehat{f_k} = \frac{1}{2\pi\Delta t} \sin^{-1}[Im(W_k)]} \tag{2.10}$$

Where Im( ) stands for imaginary part of the quantity.

2.1.2.1. Simulation results of complex LMS algorithm:

The complex LMS algorithm is applied to estimate the fundamental frequency from the sampled value of three phase voltage signal. The sampling frequency used for the computer implementation is 1 KHz and the influencing parameters of the algorithm are $\mathbf{0.0001} < \mu < \mathbf{0.18}$, $\mathbf{p_{initial}} = \mathbf{0}$, $\mathbf{r} = \mathbf{0.99}$, $\boldsymbol{\gamma} = \mathbf{0.01}$ and $\boldsymbol{\lambda}$ value changes according to sampling instant.

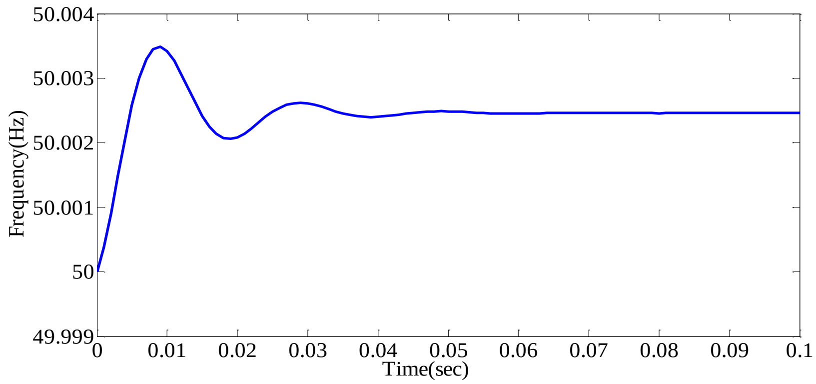


Fig.2.2. Estimation of 50.002 Hz frequency initialization by 50Hz in presence of noise.

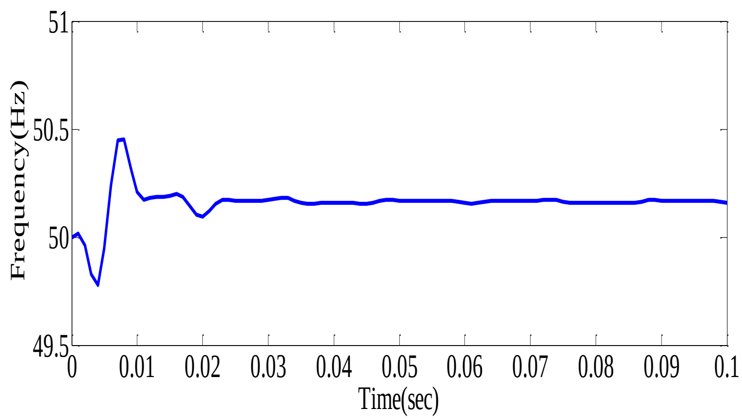


Fig.2.3. Estimation of 50.5 Hz frequency in presence of harmonics

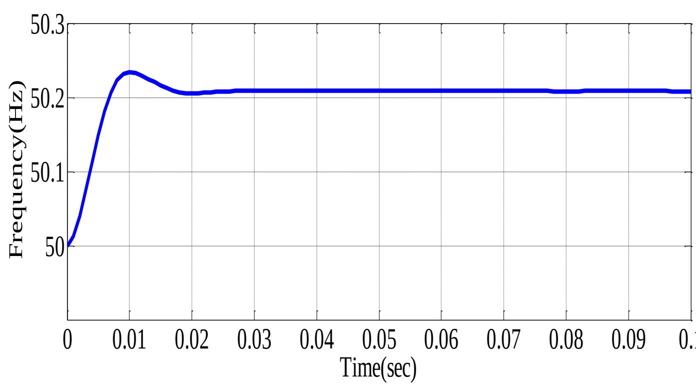


Fig.2.4. the 50.2 Hz frequency estimation under unbalanced amplitude

Case 1: In Presence of Noise: Presence of noise in the signal affects the performance of the frequency estimation technique. Here a fundamental signal of 50 Hz frequency with signal to noise ratio (SNR) of 40dB is considered and applied to the proposed algorithm and its performance is shown fig.2.2. It is observed that after 2 cycles it acquired steady state condition at 50.002Hz.

Case 2: In Presence of Harmonics: as we known electric system are very influenced by the harmonics which is the multiple of fundamental frequency that affects the convergence of the filter. Fig.2.3 presented the 50.2Hz frequency estimation in presence of 5th harmonics.

Case 3: In Presence of Unbalance Condition: the proposed algorithm works satisfactory in presence of unbalance in magnitudes of the three phase signal, which is shown on the fig.2.4 For the value of $\mathbf{V_a = 1}$p.u.**,** $\mathbf{V_b = 1.1}$p.u**.** and $\mathbf{V_c = 0.9}$p.u.**,** the estimated frequency 50.2Hz is obtained after 1 to 2cycles.

2.1.3. Nonlinear least square based frequency estimation:

The estimation of frequency using nonlinear least square is obtained by minimizing the squared error between the assumed signal model and actual signal. For the model, $\mathbf{Z}$ **(k)** is the actual signal, $\emptyset(\mathbf{k})$ is the system structure matrix and $\boldsymbol{\theta}(\mathbf{K})$ is the vector for unknown parameters that are estimated.

The unknown parameter can be obtained by

$$\widehat{\boldsymbol{\theta}_{\mathbf{LS}}} = [\emptyset(\mathbf{k})\emptyset(\mathbf{k})^{\mathbf{T}}]^{-\mathbf{1}}\emptyset(\mathbf{k})\mathbf{Z}(\mathbf{k}) \qquad \mathbf{(2.11)}$$

In this approach, a periodic voltage signal z (k) is represented in the form of Fourier series as follows

$$\mathbf{z}(\mathbf{t}) = \mathbf{A_0} + \sum_{\mathbf{n=1}}^{\infty}(\mathbf{A_n cosn\omega_0 t} + \mathbf{B_n sinn\omega_0 t}) \qquad \mathbf{(2.12)}$$

Where $\sqrt{(\mathbf{A_n^2} + \mathbf{B_n^2})}$ is the magnitude of the nth harmonic component and $\boldsymbol{\omega_0}$is the fundamental frequency in radians per second. Since $\mathbf{z}(\mathbf{t})$ does not contain any dc component, $\mathbf{a_0 = 0.}$ In practical, the number of harmonics used in the model has to be finite, say N. In power system half-wave symmetry waveform are common, these waveform does not contain even number and triplen harmonics. Hence, $\{\mathbf{1, 5, 7, 11, \ldots.. n_h}\}$. Let $\mathbf{N_a}$ is the total number of harmonics present.

Assume that $\boldsymbol{\omega_0}$ is known for solving $\mathbf{A_n}$ and $\mathbf{B_n}$ so we assume that $\mathbf{z(t)}$ has **M** uniformly sampled points. This leads to **M** set of equation as follows

$$\mathbf{z(t_k) \approx \sum_{n=1}^{N}(A_n cosn\omega_0 t_k + B_n cosn\omega_0 t_k)} \tag{2.13}$$

Equation (13) can be represented in matrix notation as follows

$$\mathbf{PX = z} \tag{2.14}$$

Where,

$$\mathbf{P = [P_a \quad P_b]}$$

$$\mathbf{P_a}=\begin{bmatrix} cos\omega_0 t_1 & cos5\omega_0 t_1 & \cdots & cosn_h\omega_0 t_1 \\ cos\omega_0 t_2 & cos5\omega_0 t_2 & \cdots & cosn_h\omega_0 t_1 \\ \vdots & \vdots & \vdots & \vdots \\ cos\omega_0 t_M & cos5\omega_0 t_M & \dots & cosn_h\omega_0 t_M \end{bmatrix}_{M\times N_a}$$

$$\mathbf{P_b} =\begin{bmatrix} \mathbf{sin\omega_0 t_1} & \mathbf{sin5\omega_0 t_1} & \cdots & \mathbf{sinn_h\omega_0 t_1} \\ \mathbf{sin\omega_0 t_2} & \mathbf{sin5\omega_0 t_2} & \cdots & \mathbf{sinn_h\omega_0 t_2} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{sin\omega_0 t_M} & \mathbf{sin5\omega_0 t_M} & \dots & \mathbf{sinn_h\omega_0 t_M} \end{bmatrix}_{\mathbf{M\times N_a}}$$

$$\mathbf{X= [a_1 \quad a_5 \dots \quad a_{nh} b_1 \quad b_5 \dots \quad b_{nh}]_{1\times 2N_a}} \tag{2.15}$$

And

$$\mathbf{Z= [z(t_1) \quad z(t_2) \dots \quad z(t_M)]^T_{1\times M}} \tag{2.16}$$

Here number of equation is $2N_a$ and we have to solve the M number of sample along with $2N_a$ equations. But measurement error and additive noise are present then over determined system of equations will give a better solution, leading to an over determined set of equations (i.e. M > $2N_a$).

The least squares solution X is written

$$\mathbf{X \approx (P^T P)^{-1} P^T z} \tag{2.17}$$

If $\boldsymbol{\omega_0}$is not known then **P** is unknown and the linear least square problem becomes nonlinear one. Even though $\mathbf{2N_a}$**+1** unknowns are present ,the linearly entering $\mathbf{2N_a}$ amplitude variables can be eliminated, resulting in a 1-D nonlinear least squares problem. Eliminating the linear variables is accomplished by substituting (2.17) in (2.14)

$$\mathbf{P(P^T P)^{-1} P^T z \approx z} \tag{2.18}$$

The error vector e is given by

$$\mathbf{e=[I - P(P^T P)^{-1} P^T]} \tag{2.19}$$

Note that e is a function ofw_0. The value of w_0 that minimizes $\|e\|_2^2$ is taken as the estimated frequency. Since we have only one parameter a 1-d search is enough to locate the minimum the typical behaviour of the error norm as a frequency. The frequency at which the minimum occurred is nearly coinciding with the true frequency.

2.1.3.1. Simulation result for nonlinear LS algorithm:

The robustness of the 1-d search can be decided on the accuracy of the algorithm. the typical behaviour of the error norm as a frequency is shown in the below figure where the search is carried out by varying f_0 in the steps of .1Hz .the frequency at which the minimum error occurred is nearly coincide with the true frequency. For the estimation of frequency here we considered a voltage waveform having 10% total harmonics and additive random noise which was given below:

$$v(t) = \sin(\omega_0 t) + 0.0856\sin(5\omega_0 t) + 0.0428\sin(7\omega_0 t) + 0.306\sin(11\omega_0 t) + 0.0183\sin(13\omega_0 t)$$

The sampling frequency used for the computer simulation was 3.2 KHz and the fundamental frequency is 50Hz. As we known, in power system half wave symmetry waveform are very common, so this waveform do not contain any even numbered of harmonics and triplen harmonics. Triplen harmonics are the 3rd and multiple of third harmonics. For the above sampling frequency, maximum 32 harmonics are taken into consideration without any type of aliasing, which is adequate for the application of any power system environment. By taking the given voltage signal we calculated the frequency by using nonlinear least square method varying the voltage signal from 48.5 to 51.5Hz.

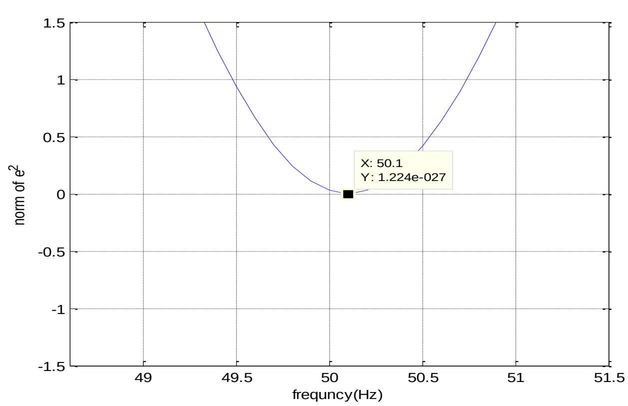


Fig.2.5. Illustration of the variation of $\|e\|_2^2$ as a function off_0 , when$f_0$varied from 48.5Hz to 50 Hz.

The simulation result shown that minimum error obtained at 50.1Hz which is the estimated frequency.

2.1.4.Power system frequency estimation using RLS method:

To improve the least square algorithm performance, many extension and modification are done in literature. To enhanced performance gained through this modification are faster convergence, less deviations, recursive calculation and consistency of estimation. This algorithm is consists of extra three step calculation in each iteration;

$$\mathbf{H}^{-1}(\mathbf{k}+\mathbf{1})=\mathbf{H}^{-1}(\mathbf{k})+\emptyset^{\mathbf{T}}(\mathbf{K}+\mathbf{1})\emptyset(\mathbf{k}+\mathbf{1})$$

$$\mathbf{K}\ (\mathbf{k+1})=\mathbf{H}\ (\mathbf{k+1})\emptyset^{\mathbf{T}}(\mathbf{k}+\mathbf{1})$$

$$\boldsymbol{\theta}(\mathbf{k}+\mathbf{1})=\boldsymbol{\theta}(\mathbf{k})+\mathbf{K}\ (\mathbf{k+1})\ [\mathbf{Z}\ (\mathbf{k+1})-\emptyset(\mathbf{k}+\mathbf{1})\boldsymbol{\theta}(\mathbf{k})] \tag{2.20}$$

The above equations are initialized by taking some initial values for the estimate at instants $\mathbf{k}$, $\boldsymbol{\theta}(\mathbf{k})$ and $\mathbf{H}$. For H=$\boldsymbol{\alpha}\mathbf{I}$, where α is a large number and I is the identity matrix of order n×n, where n is the number of parameters to be estimated. For RLS algorithm based frequency estimation the extra three steps of equation (2.20) is added after equation (13) as given in the recursive least square algorithm but here the unknown parameter vector is updated at each instant and initially it is considered as zero vector.

2.1.4.1. Simulation result of nonlinear RLS:

The simulation results of nonlinear RLS is same as the LS only difference between the two algorithms is LS is a batch process and RLS is an iterative process. Here the minimum frequency obtained at 50.1Hz which is an estimated frequency for this algorithm. In practically the characteristic of nonlinear RLS is much more attractive in connection with using the algorithm in online. The result of RLS algorithm is shown below having variation of frequency from 48.5Hz to51.5Hz with the difference of 0.1Hz.It is an online process so we can calculate the vector for unknown parameter in each step which help us to find the error in each sample. The simulation diagram is shown in fig.2.6.

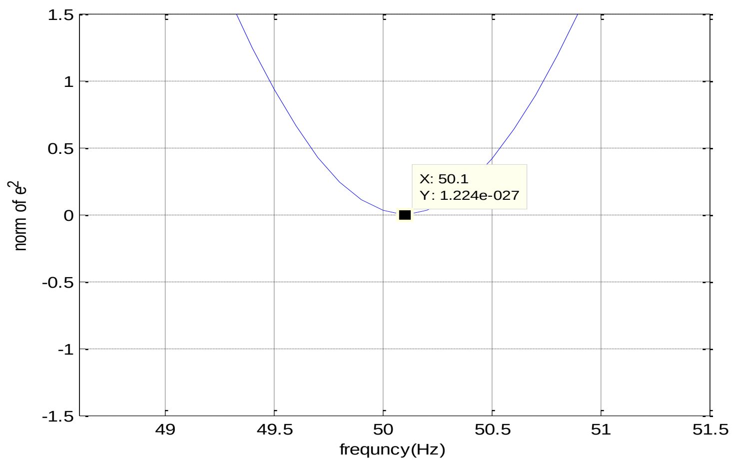
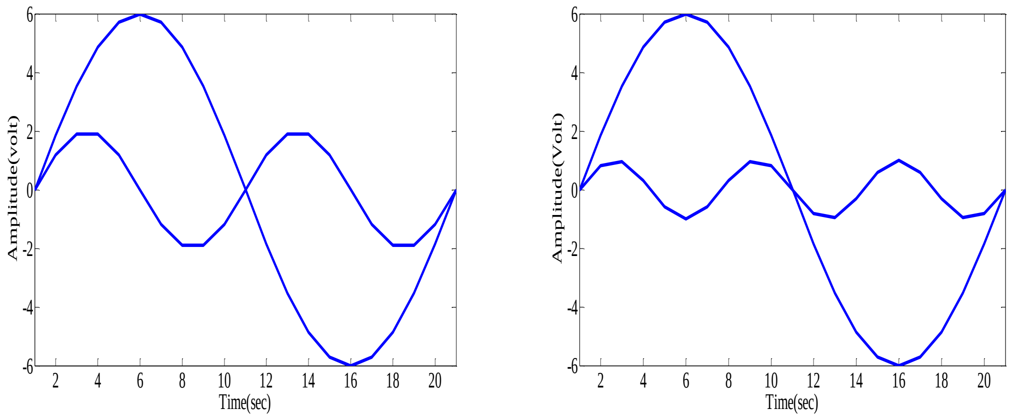


Fig.2.6. estimation of f_0 which is derived from $||e||_2^2$ where $f_0$varied from 48.5Hz to 50 Hz.

2.2 Power system harmonics estimation:

2.2.1 Introduction:

In recent decade, periodic distortion of voltage and current waveforms is not desirable in electrical network due to increase in nonlinear load and time varying device. Because of this nonlinear load or device, the voltage and current waveform contains sinusoidal component other than the fundamental frequency which is known as the harmonics. Harmonics are the multiple of fundamental frequency. The second harmonic is the component with frequency two times the fundamental (100 0r 120 Hz) and so on. As shown in Fig.2.7 harmonic distortion can be considered as a sort of pollution of the electric system which causes problems if the sum of the harmonic currents exceeds certain limits.The quality of power factor decreases due to presence of harmonics which results several problem related to power system operation, protection and mitigation. The major components of producing harmonics in commercial and industrial power system are the increasing use of nonlinear load like diode and thyristor rectifier, arc furnace, printer and uninterruptible power supplies (UPS) etc. hence it is very much tedious to calculate the harmonics from distorted waveform. So fact and approximate calculation for amplitude and phase of fundamental frequency are needed. The most commonly used classical estimation technique is based on the fast Fourier transform (FFT). Beside this there is several extension and improvements has been published in this area [12-16]. However FFT based algorithm are good in noise but having a problem of aliasing and spectral effect. In this field, kalman filtering [17, 18] is one of the robust methods for estimation of magnitude of harmonics but this kalman filtering fails to track any dynamics changes in measured signal. This section represented the calculation of amplitude and phase of fundamental frequency of voltage or current signal using recursive method. Here we obtained better results in recursive method as compare to LS.



(a) (b)

Fig. 2.7 A sinusoidal waveform with fundamental frequency 50 Hz and its harmonics: (a) second (100 Hz); (b) third (150 Hz)

2.2.2 Harmonics estimation problem:

Let as consider a signal with fundamental frequency $\boldsymbol{\omega_0}$ and its harmonics are the multiples of this frequency. Hence the signal can be written as

$$\mathbf{v(t) = A_1 sin(\omega_0 t + \varphi_1) + A_2 sin(2w_0 t + \varphi_2) + \cdots + \varepsilon(t)} \quad \textbf{(2.21)}$$

Where the amplitude of the ith harmonic is $\mathbf{A_i}$and$\boldsymbol{\varphi_i}$ is its phase and $\boldsymbol{\varepsilon}(\mathbf{t})$ is the additive noise. For the estimation of fundamental sinusoidal the above signal will be

$$\mathbf{v(t) = A_1 sin(\omega_0 k + \varphi_i) + \eta(t)} \quad \textbf{(2.22)}$$

For simple calculation, this equation can be modified for the purpose of estimation

$$\mathbf{v(t) = [sin(\omega_0 t) \quad cos(\omega_0 t)] \begin{bmatrix} \alpha \\ \beta \end{bmatrix} + \eta(t)} \quad \textbf{(2.23)}$$

or in standard form

$$\mathbf{v(t) = \phi(t)\theta + \eta(t)} \quad \textbf{(2.24)}$$

Where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are the parameters to be estimated and is given by

$$\mathbf{\alpha = A_1 cos\varphi_1}$$
$$\mathbf{\beta = A_1 sin\varphi_1} \quad \textbf{(2.25)}$$

The actual required parameters are amplitude and phase which can given below

$$\mathbf{A_1} = \sqrt{\boldsymbol{\alpha}^2 + \boldsymbol{\beta}^2}$$

$$\boldsymbol{\varphi_1} = \mathbf{tan}^{-1}\frac{\boldsymbol{\beta}}{\boldsymbol{\alpha}} \tag{2.26}$$

For multiple harmonics signal $\boldsymbol{\phi}(\mathbf{t})$ and $\boldsymbol{\theta}(\mathbf{t})$ matrix can be represented as follows

$$\boldsymbol{\phi}(\mathbf{t}) =$$

$$\begin{bmatrix} \sin(\omega_0 t_1) & \cos(\omega_0 t_1) & \sin(2\omega_0 t_1) & \cos(2\omega_0 t_1) & \cdots & \sin(n\omega_0 t_1) & \cos(n\omega_0 t_1) \\ \sin(\omega_0 t_2) & \cos(\omega_0 t_2) & \sin(2\omega_0 t_2) & \cos(2\omega_0 t_2) & \cdots & \sin(n\omega_0 t_2) & \cos(n\omega_0 t_2) \\ \sin(\omega_0 t_k) & \cos(\omega_0 t_k) & \sin(2\omega_0 t_k) & \cos(2\omega_0 t_k) & \cdots & \sin(n\omega_0 t_k) & \cos(n\omega_0 t_k) \end{bmatrix}$$

And

$$\boldsymbol{\theta}(\mathbf{t}) = [\boldsymbol{\alpha}_1 \quad \boldsymbol{\beta}_1 \quad \boldsymbol{\alpha}_2 \quad \boldsymbol{\beta}_2 \quad \cdots \quad \boldsymbol{\alpha}_n \quad \boldsymbol{\beta}_n] \tag{2.27}$$

The equation (2.27) gives the idea for calculating vector for unknown parameter in presence of additive noise.

2.2.3. Simulation results of harmonics estimation:

Recursive algorithm is applied to estimate the fast and accurate estimation of amplitude and phase from a distorted signal. The distorted signal is constructed by using the values used by various authors in this area. The sampling frequency used for the estimator problem is 3.2kKz and fundamental frequency is 50Hz. For estimation of harmonics magnitude we consider the above voltage signal with 13^{th} harmonics and additive random noise. The voltage signal is given below

$$v(t) = \sin(\omega_0 t) + 0.0856\sin(5\omega_0 t) + 0.0428\sin(7\omega_0 t) + 0.306\sin(11\omega_0 t) + 0.0183\sin(13\omega_0 t)$$

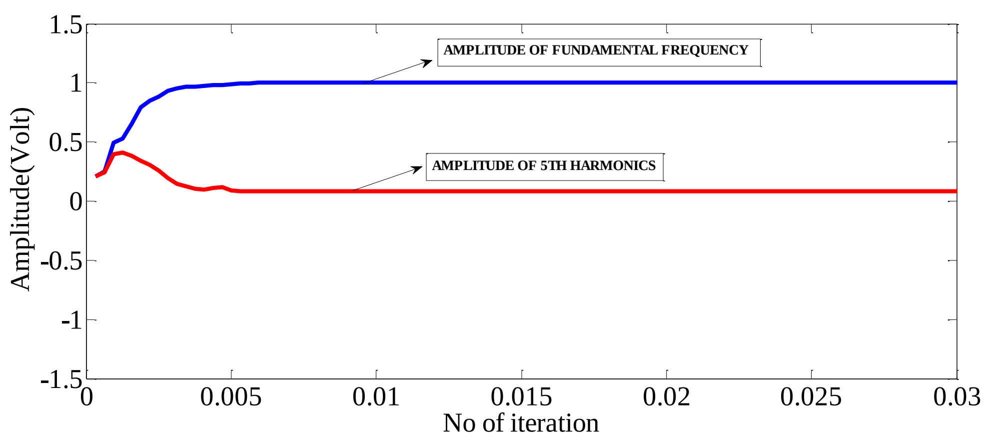


Fig.2.7. Amplitude of fundamental frequency and fifth harmonics of signal v (t).

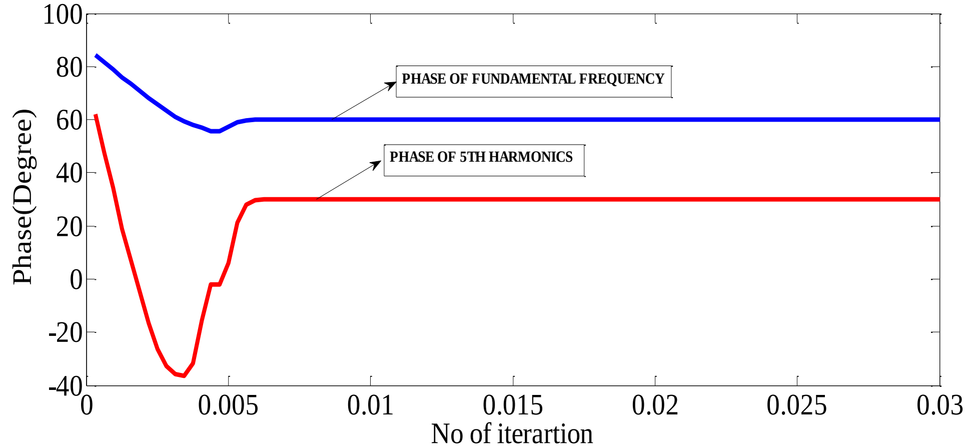


Fig.2.8. Phase of fundamental frequency and fifth harmonics of signal v (t)

Fig.2.7 and 2.8 shows the results for estimated amplitude and phase for the distorted signal. The actual value of for the amplitude of 1st and 2nd harmonics is 1 and 0, 0856 respectively and the phases are 60 and 30 degree. The results show almost exact estimation. Here we consider one case were we taken only odd harmonics with absences of even and triplen harmonics. By using this algorithm we can estimate the all the harmonics presented in the distorted signal.

2.3 Summary of the chapter:

This chapter deals with the two main aspects. One section is for estimating power system frequency and other is for calculating harmonics. First section consists of three algorithms for calculating the frequency which are complex LMS, nonlinear LS and RLS algorithm. Complex LMS algorithm calculates fundamental frequency in presence of noise, harmonics and unbalance condition of amplitude from a three phase voltage signal whereas nonlinear LS and RLS algorithms are estimating frequency in presence of random noise by varying the frequency from some allowable range. The complex LMS algorithm is simple and flexible method for calculating the frequency and it overcome the poor convergence rate of LMS algorithm. Nonlinear LS and RLS methods are probably good for estimating frequency. Second part of the chapter contains calculations of harmonics in presence of additive noise by using recursive method. Here we calculated the magnitude of harmonics which is nearly equal to magnitude of supply voltage's harmonics. The entire above algorithm are verified by the simulation results.

CHAPTER-3

EXPERIMENTAL SETUP

3.1 Introduction:

Frequency and harmonics are the two operating components in power system. Frequency of a system remains constant if the sum of loads and losses is equal to the generation of the system. If there is any up and down in between the sum of load and losses and generation power of device then there is a fluctuation of frequency in the system which may have adverse effect in system. Similarly if there is a presence of harmonics other than fundamental frequency, then it is also having some negative impact in the system. To avoid this entire problem, some of frequency and harmonics estimation techniques are discussed in the previous chapter. In this chapter, the proposed technique was implemented by using DSO (Digital Storage Oscilloscope) based general proposed hardware which is very common instrument in electrical and electronics laboratory. The discrete data of distorted signal was got from DSO and that was evaluated by the above technique. And their experimental results are discussed below.

3.2 Implementation:

Performance of different algorithm has been evaluated on MATLAB simulation. It would be very much interesting to have the same on experimental setup. Fig 3.1 and 3.2 show the block diagram and laboratory prototype. At first we collected the data from the experimental set up and this data is obtained from the normal working day supply. The components present in the hardware are rectifier load (diode bridge rectifier), DSO and computer with PC communication software.

Specifications of the Instruments used are:

SUPPLY: 220 volt single phase supply is given to the experimental set up.

AUTOTRANSFORMER: a single phase autotransformer with specification maximum load 15 Amps, KVA 4.05, input 240V and output 260V @ 50/60 Hz frequency. In normal power supply there is fluctuation in voltage which may be damage the set up. So we connected autotransformer to provide constant voltage supply to the set up.

ISOLATION TRANSFORMER: Two isolation transformers with primary voltage 230V and secondary voltage 115 et 115 is connected to the set up. As we know, the voltage signal from the supply and the outside world can be too noisy and dangerous to measure directly. Hence the isolation transformer manipulates a signal into a form which is suitable for the setup. It passes the voltage signal from the supply to the measuring device without physical connection and it is having 1:1 turn ratio of winding. Here one isolation transform is connect to the supply and other is connected with the DSO to provide isolation with the direct supply

NONLINEAR LOAD: a single phase diode bridge rectifier with a 100 Ω resistor in series with a 250mH inductor at the dc side. In a nonlinear load, current is not proportional to voltage, which results distortion of the voltage signal. For creating the distortion, we connected a rectifier load with the supply.

DIGITAL STORAGE OSCILLOSCOPE (Tektronix with kit no TPS2014): Band Width-100 MHz, Sample rate-1 GS/s, 4 channel, Record length-2500 data points, PC Connectivity- 9 pin female port and PC Communication software. In DSO we can capture the signal and store the signal in the form of discrete form into the computer with the help of pc communication software. PC communication software is available with respective DSO.

VOLTAGE PROBE: A 10X voltage probe is used to capture the distorted signal from the experimental set up to the digital oscilloscope.

COMPUTER: 1.46 GHz CPU and 1GB RAM, desktop computer. It stores the data in the form of discrete form with the help of pc communication software, which is captured by the DSO. The sampling time in this case is fixed at 0.04ms. The above data is used to estimate the frequency and harmonics with the help of the above described algorithm except the complex LMS algorithm. The simulation results are shown below.

Here the software part is divided into two sections. One section is the pc communication software and other section is MATLAB coding. In first section, we captured the generated data in discrete format and with the help of MATLAB coding, the discrete data is used to calculate the fundamental frequency and harmonics as shown in the below figure.

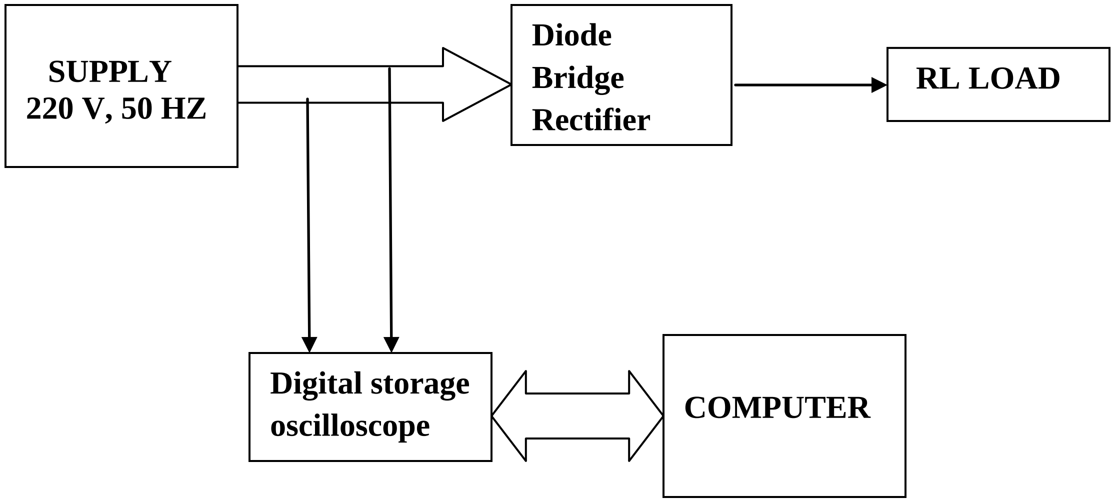


Fig.3.1 block diagram of experimental set up

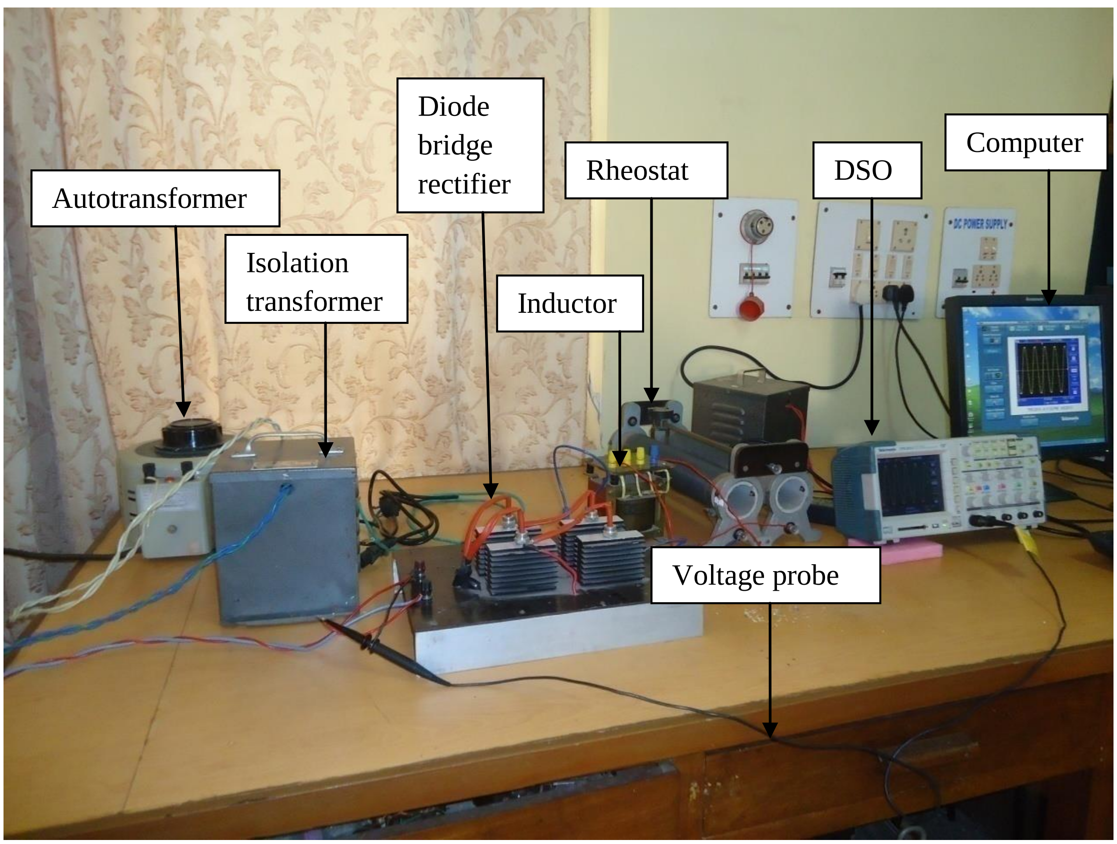


Fig 3.2 Photograph of laboratory setup

3.3. Performance evaluation:

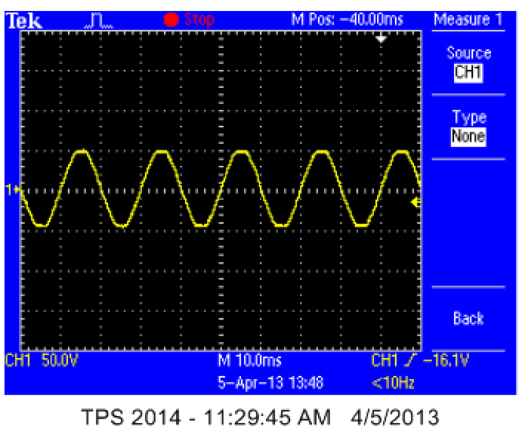
After implementing the above algorithm, it was evaluated to determine its measurement accuracy and speed. The test signal is generated by the experimental set up which was analyzed by the above algorithm that was shown below.

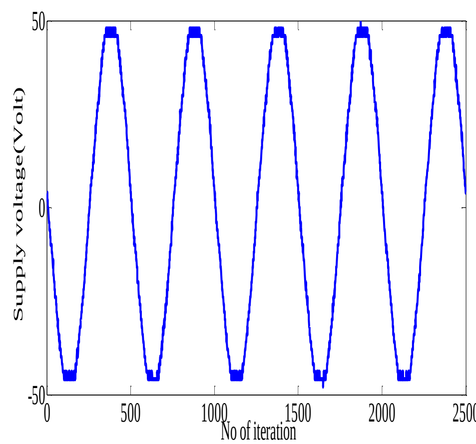
Simulation results:

The simulation results were carried out by considering two cases, one case with low voltage and minimum inductor load and other with maximum voltage and maximum inductor load.

(A) Considering low voltage:

Here the supply voltage is 33volt after connecting the nonlinear load the maximum peak voltage shown by the DSO is 48volt and the frequency remain the same 50Hz. The experimental results of the above case are shown below.





(a) (b)

Fig.3.3 (a) distorted waveform generated by the DSO and (b) distorted waveform diagram in MATLAB by taking low voltage and minimum load

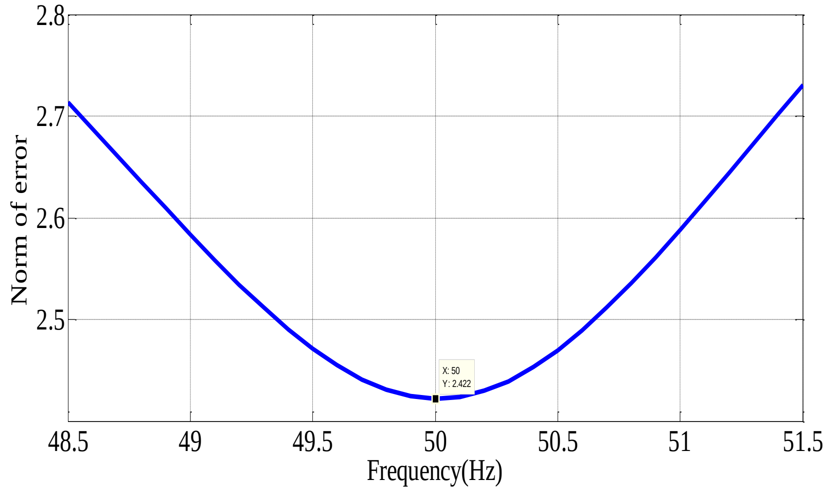
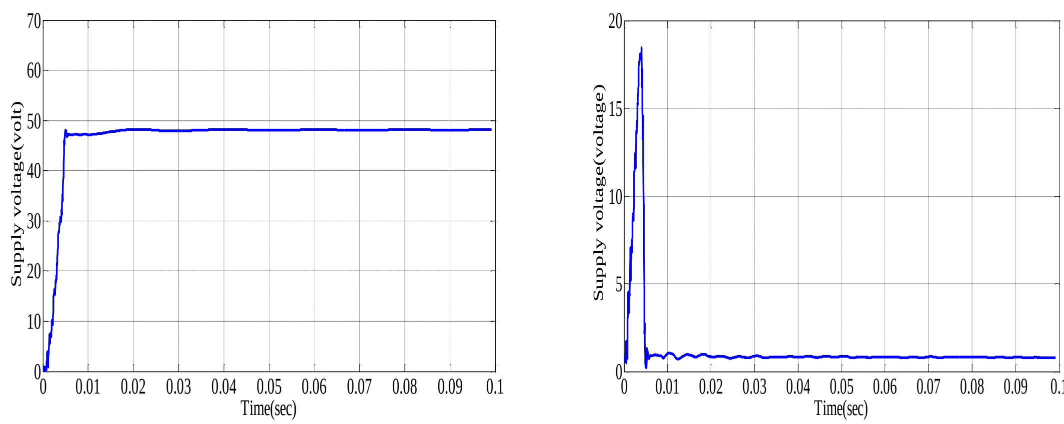


Fig 3.4 Estimation of frequency from the generated low voltage distorted waveform

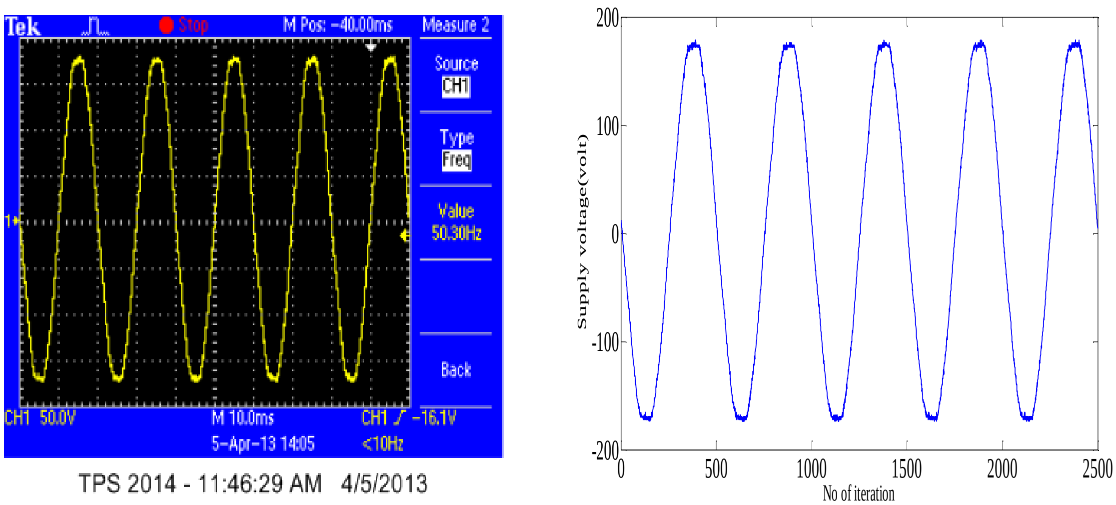


(a) (b)

Fig.3.5. (a) Amplitude of fundamental frequency and (b) 5^{th} harmonics of the generated low voltage distorted waveform using nonlinear RLS algorithm

(B) Considering maximum voltage:

Here the supply voltage is 120 volt and after connecting to the nonlinear load the maximum peak voltage shown by DSO is 178volt and the frequency becomes 50.30Hz. DSO is having 50volts per division screen so it can capture the maximum peak voltage of 178 volt. For this case, peak to peak voltage is 352 volts and the negative peak voltage is 174 volt. Due to the maximum supply voltage, the distortion of the signal is not clearly visible but the nonlinear load injecting the nonlinearity to the experimental set up so there was a deviation of frequency from its nominal value which can be observed by the DSO. The experimental results of the distorted signal are shown below.



(a) (b)

Fig.3.6 (a) distorted waveform generated by the DSO and (b) distorted waveform diagram in MATLAB by taking high voltage and 250mH inductor load

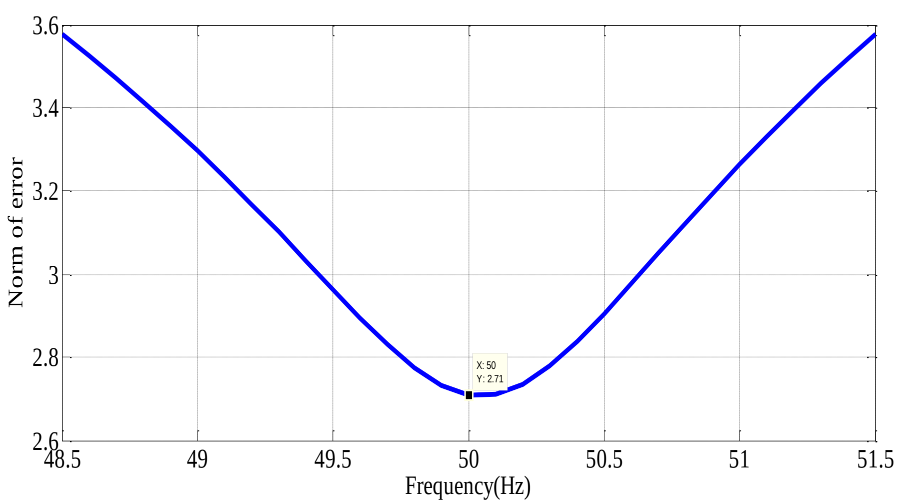
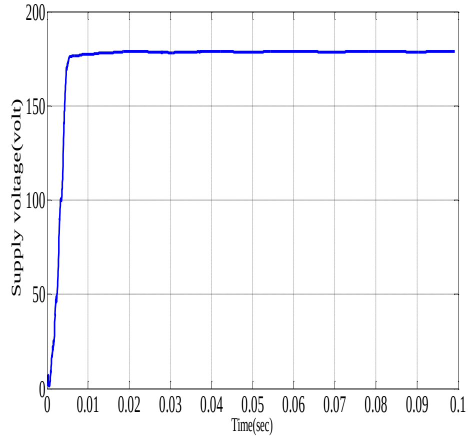
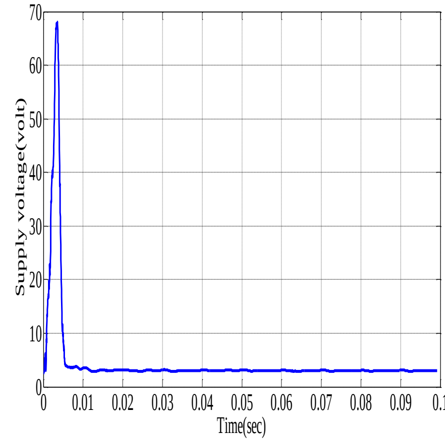


Fig 3.7 Estimation of frequency from the generated high voltage distorted waveform using nonlinear LS algorithm





(a) (b)

Fig.3.8 Amplitude of fundamental frequency and (b) 5th harmonics of the generated high voltage distorted waveform using nonlinear RLS algorithm

Discussion of experimental results:

Here we consider a diode bridge rectifier with 100 ohm resistor and inductor of 50-250mh as a nonlinear load, which provided the nonlinearity to the supply voltage. In the above section, we discussed experimental set up results by considering two conditions one by taking low voltage with minimum inductor load and another with maximum voltage which can capture by the DSO and maximum inductor load. In first case, due to the nonlinear load the peak portion of the waveform is flatter which show the distortion of the sinusoidal wave. Because of the low voltage, the flatter portion of the distortion voltage can be visible. As we increases the voltage the flatness of the waveform decreases. The only difference between the two cases was the flatness of the waveform which was prominent in the first case. In case of low voltage there is no deviation of frequency but in high voltage there is a deviation of frequency from 50 to 50.3Hz.So as we increases the voltage and the inductive load, there was a deviation of frequency from its standard value. The distorted waveform figure are provided above in fig3.3 to fig 3.8.by collecting data from the distorted waveform, that was implemented in the described algorithm which was given frequency nearly equal to the estimated frequency and amplitude of fundamental frequency. From the above simulation results it shows that in first case the minimum norm of error occurred at 50Hz frequency which is an estimated frequency of that signal. In that case there was no deviation of frequency but in case of second one the minimum error occurred from 50 to 50.1Hz due to the deviation of frequency. In case of harmonics estimation, we estimated the maximum peak voltage of fundamental frequency after 0.01sec.In the generated signal, there is no higher harmonics other than the fundamental frequency, therefore when we calculated the 5^{th} harmonics it raised to some extend after that it became to zero.

3.4 Chapter summary:

This chapter contains the information about the experimental setup which is proposed by considering the above described algorithm. To evaluate the robustness of the algorithm we proposed this set up. This setup provided the data of distorted waveform which is further used to estimate the power system parameters. The nonlinear LS and RLS work well in this field for calculating the frequency and harmonics magnitude respectively. But one drawback is that the experimental setup not provided enough harmonics to the waveform for that we have to use high value inductor and resistor and we can replace the diode rectifier by thyristor rectifier.

CHAPTER-4

4.1Conclusions:

Based on the studies conducted in this paper, the following conclusion may be drawn about the well-known above algorithm for estimation of power system parameter.

- The LMS-based approach using sampled values of three-phase voltage signals is simple in its formulation. The simulation results show that the accuracy and speed of estimation is satisfactory even in the presence of noise/harmonics and during in presence of unbalance amplitude this method works well.
- Flexibility of nonlinear 1-d search grid of least square can be decided based on the accuracy desired; the frequency at which the minimum value occurred is coincided with the true value of frequency.
- Simulation result of RLS algorithm is same as the LS algorithm but in RLS we updated the frequency at each sample. Hence nonlinear RLS is an online estimation method.
- The results obtained from the harmonics estimator problem using recursive algorithm are fairly closed to the true value of the distorted signal.
- From the experimental set up, we verified the above algorithm and the nonlinear LS and RLS stand well in this experimental work. in experimental setup, nonlinear LS worked well for estimation of frequency whereas the nonlinear RLS worked well for the estimation of harmonics magnitude.

4.2 SUGGESTIONS FOR FUTURE WORK:

- Hardware implementations can be done on D-space for evaluating the above algorithms.
- Hardware implementation on FPGA for calculation of power system parameters.
- Hardware implementation on DAQ for calculation of power system parameters.

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